

## Graph Theory Homework 2

Due: 4 June 2019 at 3:59pm as a PDF on Submitty

**v1.1:** Updated 30 May 2019

1. We have a weakly connected loopless directed graph  $D$  with  $\forall v \in V(D) : d^+(v) = 1$  and  $n = |V(D)| \geq 2$ . Answer the following in terms of  $n$ , prove or otherwise justify your responses:
  - (a) What are the maximum and minimum number of cycles that  $D$  can have in terms of  $n$ ?
  - (b) What is the maximum number of cycles if  $D$  isn't weakly connected?
  - (c) What is the maximum number of cycles if  $D$  is neither weakly connected nor loopless?
2. Consider integer sequence  $S = \{1, 2, 1, 1, 4, 3\}$ . Construct a realization of a graph  $G$  if we consider  $S$  to be a graphic sequence. Construct a realization of a tree  $T$  if we consider  $S$  to be a Prüfer Code with vertex set  $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$ .
3. Assume a simple undirected and connected graph  $G$  can be realized by the below graphic sequence. Prove whether or not  $G$  has a cut edge.  
 $S = \{8, 8, 6, 6, 4, 6, 4, 2, 4, 2, 4, 6, 8, 10, 10, 10, 8, 4, 2, 8, 4, 6, 2, 2, 2, 2, 8, 2, 2, 2\}$
4. Use an extremal argument to prove that in every tree  $T$ , any two maximum length paths *of equal length* (**v1.1**) must intersect (have a common vertex  $v$ ).
5. Use an extremal argument to prove that every simple graph with at least two vertices contains two vertices of the same degree.
6. Let  $G$  be a graph which has fewer edges than vertices ( $|E(G)| < |V(G)|$ ). Use induction to prove that at least one connected component of  $G$  is a tree.